Variations on the Gauge Sector of the Electroweak Model

J. PESTIEAU

Institut de Physique Théorique
Université catholique de Louvain,
BELGIUM

11 August 2009

Abstract

Starting from a 40 year old proposal, new relations between alpha, the fine structure constant, Z and W masses are proposed.

I. Forty years ago\(^{(1)}\), it has been proposed the following determination of Z and W masses

\[
\bar{m}_Z = \frac{A_0}{\sin \bar{\theta} \cos \bar{\theta}}
\]

\[
\bar{m}_W = \frac{A_0}{\sin \bar{\theta}}
\]

with\(^{(2)}\)

\[
A_0 = \left( \frac{\pi \alpha}{\sqrt{2} G_F} \right)^{1/2} = 37.28057(8) \text{ GeV}
\]

\[
\sin \bar{\theta} = \sqrt{\frac{3}{14}}
\]

(The weak angle \( \bar{\theta} \) in Eqs (1) and (2) is the complementary angle of \( \theta \) defined in Ref (1) : \( \theta + \bar{\theta} = \frac{\pi}{2} \)).

Then

\[
\bar{m}_Z = 90.85560(19) \text{ GeV}
\]

1. jean.pestieau@uclouvain.be
\[ m_W = 80.53524(17) \text{ GeV} \]  

(6)

to be compared with their experimental values\(^{(2)}\):

\[ m_Z = 91.1876(21) \text{ GeV} \]  

(7)

\[ m_W = 80.398(25) \text{ GeV} \]  

(8)

Let us present variations on Eqs (1) and (2).

II. It is amusing to consider the following simple parametrizations:

A.

\[ m_Z = \bar{m}_Z (1 + \frac{\alpha}{2}) = 91.18750(19) \text{ GeV} \]  

(9)

\[ m_W = \bar{m}_W (1 + \frac{\alpha}{2})^{-1/2} = 80.38871(17) \text{ GeV} \]  

(10)

B.

\[ m_Z = \bar{m}_Z \left( \frac{\cos \bar{\theta}}{\cos \theta_W} \right)^{2/3} = 91.18757(19) \text{ GeV} \]  

(11)

\[ m_W = \bar{m}_W \left( \frac{\cos \theta_W}{\cos \theta} \right)^{1/3} = 80.38868(17) \text{ GeV} \]  

(12)

with\(^{(3)}\)

\[ \alpha = \frac{e^2}{4\pi} = [137.035999084(51)]^{-1} \]  

(13)

\[ \cos \theta_W \equiv \frac{m_W}{m_Z} \]  

(14)

We used value of \( \cos \theta_W \) obtained from the empirical relation\(^{(4)}\)

\[ 1 - \tan^2\left( \frac{\pi}{4} - \theta_W \right) = 3e. \]  

(15)

Note that

\[ 1 - \tan^2\left( \frac{\pi}{4} - \theta_W \right) = \frac{4 \sin \theta_W \cos \theta_W}{(\sin \theta_W + \cos \theta_W)^2}. \]  

(16)

III. To make contact with a well known parametrization\(^{(2)}\)

\[ m_Z = \frac{A_0}{\sin \theta_W \cos \theta_W} \left( \frac{1}{1 - \Delta r} \right)^{1/2} \]  

(17)

we write Eq. (11) as

\[ m_Z = \frac{A_0}{\sin \theta_W \cos \theta_W} \left( \frac{\sin \theta_W}{\sin \bar{\theta}} \right) \left( \frac{\cos \theta_W}{\cos \theta} \right)^{1/3}. \]  

(18)
Then
\[
\frac{1}{(1 - \Delta r)^{1/2}} = \left( \frac{\sin \theta_W}{\sin \theta} \right) \left( \frac{\cos \theta_W}{\cos \theta} \right)^{1/3}
\] (19)
in the current context.

IV. It is interesting to note the following empirical formula(4):

\[
m_Z = \frac{1}{\sin \theta_W + \cos \theta_W} \left( \frac{\cos \theta}{\cos \theta_W} \right)^{23/48} \frac{v_F}{2}
\] (20)

\[
m_Z = \frac{A_0}{\sin \theta_W \cos \theta_W} \frac{3}{4} (\sin \theta_W + \cos \theta_W) \left( \frac{\cos \theta}{\cos \theta_W} \right)^{23/48}
\] (21)

\[
m_Z = 91.18756(19) \text{ GeV}
\] (22)

where we have used

\[
A_0 = \frac{e \frac{v_F}{2}}
\] (23)

and Eqs (15-16).

V. About \( e \) and \( \alpha \)

With \( \alpha \) and \( e \) given in Eqs (13), we satisfy the following Equation(4)

\[
\frac{1}{e} - e \left[ 1 - \frac{\alpha}{4} - \left( \frac{\alpha}{4} \right)^2 - x \left( \frac{\alpha}{4} \right)^3 \right] = 3
\] (24)

when \( x = 0.430 \pm 0.365 \).

For example,

if \( x = 0.75 \), then \( \alpha^{-1} = 137.035999039 \)

if \( x = 0.50 \), then \( \alpha^{-1} = 137.035999074 \)

if \( x = 0.25 \), then \( \alpha^{-1} = 137.035999109 \)

Comparing Eqs (15) and (24), we get

\[
\tan^2 \left( \frac{\pi}{4} - \theta_W \right) = e^2 \left[ 1 - \frac{\alpha}{4} - \left( \frac{\alpha}{4} \right)^2 - x \left( \frac{\alpha}{4} \right)^3 \right]
\] (25)

(In Ref. (4), the following approximation of Eq. (25) is used : \( \tan^2 \left( \frac{\pi}{4} - \theta_W \right) = e^2 \)).

It is worthwhile to note that

\[
\frac{1}{e} - e \left[ 1 - \frac{\alpha}{4} \exp \left( \frac{\alpha}{4} \right) \right] = 3
\]

is satisfied when

\[
\alpha^{-1} = 137.035999074.
\]
REFERENCES


